

## Multiple-retrapping processes in the phase-diffusion regime of high- $T_c$ intrinsic Josephson junctions

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We report measurements of switching current distribution (SWCD) from a phase-diffusion branch (PDB) to a quasiparticle-tunneling branch (QTB) as a function of temperature in a cuprate-based intrinsic Josephson junction. Contrary to the thermal-activation model, the width of the SWCD increases and the corresponding switching rate shows a nonlinear behavior with a negative curvature in a semilogarithmic scale with decreasing temperature down to 1.5 K. Based on the multiple-retrapping model, we quantitatively demonstrate that the frequency-dependent junction quality factor, representing the energy dissipation in a phase-diffusion regime, determines the observed temperature dependence of the SWCD and the switching rate. We also show that a retrapping process from the QTB to the PDB is related to the low-frequency limit damping.

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### I. INTRODUCTION

The escape of a system trapped in a metastable state governs the reaction rate in various dynamical systems, where the escape is made by a noise-assisted hopping.<sup>1</sup> In the case of a Josephson junction (JJ), a thermal noise induces an escape of a phase particle representing the system from a local minimum of the potential well. In an *underdamped* JJ with hysteresis in the voltage-current ( $V$ - $I$ ) characteristics a single escaping event induces a switching from a zero-voltage phase-trapped state to a finite-voltage phase-rolling state, which gives a switching current,  $I_{sw}$ .<sup>2</sup> In an *overdamped* JJ without hysteresis, however, the energy of an escaped phase particle is strongly dissipated during its motion so that the particle is retrapped in another local minimum of the potential. The phase particle repeats this thermally-activated escape and retrapping process, i.e., the multiple-retrapping process in an overdamped JJ. It results in a phase-diffusion branch (PDB) with a small but finite voltage for a bias current below the critical current.<sup>3</sup>

A hysteretic JJ can also evolve into this multiple-retrapping regime if the temperature is sufficiently high.<sup>4</sup> Recently, this phase-retrapping phenomenon in JJs with hysteresis, including cuprate-based intrinsic Josephson junctions (IJJs), has been intensively studied in association with the temperature dependence of a switching current distribution (SWCD).<sup>5-7</sup> The main finding of these studies is that the retrapping process in the hysteretic JJ modifies the switching dynamics in such a way as to *reduce the width of SWCD with increasing temperature*. This SWCD behavior, in contrast to the usual thermal-activation model, has also been suggested to be caused by the enhanced dissipation due to a frequency-dependent impedance.<sup>10</sup> The impedance of the measurement lines ( $Z_L=50-100\ \Omega$ ) is the main source of the high-frequency dissipation. As the dissipation of a JJ is represented by its quality factor, previous studies have focused only on the moderately damped regime of quality factor  $\sim 5$ , where the  $V$ - $I$  curves with the SWCDs do not exhibit PDBs because the rate of the phase diffusion is too low to show a finite voltage at the temperatures studied. Thus, although the

PDB itself has been extensively studied in conventional IJ and IJJ systems,<sup>4,11-15</sup> the switching dynamics between a PDB and a quasiparticle-tunneling branch (QTB) has not been clearly resolved in terms of the quality factor.

In this paper, we report switching from the PDB to the QTB in IJJs of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  (Bi-2212) at various temperatures below  $T=4.2$  K. Temperature dependence of the switching rate  $\Gamma_S$  and the corresponding SWCD in a single IJJ are in good agreement with those estimated by the multiple-retrapping model. This study clarifies, in a quantitative manner, how the shunt impedance- and temperature-related dissipations and the corresponding frequency-dependent quality factor  $Q(\omega)$  determine the switching dynamics in a hysteretic IJJ with phase-diffusion characteristics. In the relevant frequency range the quality factor was between 1.3 and 2.4. We show that the retrapping dynamics from the QTB to the PDB is determined by the low-frequency limit damping.

### II. EXPERIMENTS

A stack of IJJs with a lateral size of  $2.5 \times 2.9\ \mu\text{m}^2$  in Bi-2212 single crystal was defined using the focused-ion-beam (FIB) process as shown in the lower inset of Fig. 1(a),<sup>16</sup> where the stack under measurements was the region bounded by dashed lines in the corresponding schematic in the upper inset of Fig. 1(a). High-intensity FIB irradiation on a single crystal is known to degrade the peripheral region by the scattered secondary-ion beam.<sup>17</sup> In the milling process, we used a relatively high ion-beam current of 3 nA, corresponding to an intensity of  $\sim 200\ \text{pA}/\mu\text{m}^2$ . This high ion-beam current reduced the interlayer tunneling critical current density down to  $\sim 8\ \text{A}/\text{cm}^2$  in  $N=12$  junctions out of the total of  $\sim 100$  IJJs in the stack, which were estimated from the number of QTBs in the  $V$ - $I$  curves of the stack (not shown). Four-terminal transport measurements [see the upper inset of Fig. 1(a)] were carried out in a pumped He<sup>4</sup> dewar with a base temperature of 1.45 K. Room-temperature  $\pi$  filters were employed and measurement lines were embedded in silver paste at cryogenic temperatures to suppress

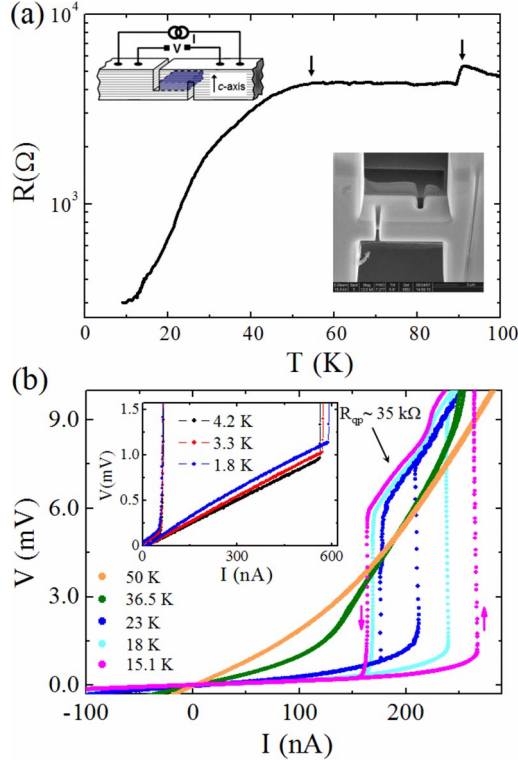


FIG. 1. (Color online) (a)  $R$  vs  $T$  curve of the sample stack, where  $T_c$  of the crystal is indicated by the right arrow. The left arrow indicates the critical temperature of the phase-diffusive junctions. Upper inset of (a): schematics for the sample and the measurement configurations. Lower inset of (a): scanning electron microscope image of the measured sample. (b)  $V$ - $I$  characteristics at various temperatures of the Bi-2212 IJJ stack. The upward and downward arrows indicate the switching and the return currents, respectively, at  $T=15.1$  K, with a resistance of the QTB,  $R_{qp} \sim 35$  k $\Omega$ . Inset of (b):  $V$ - $I$  curves of the sample at  $T=1.8$ , 3.3, and 4.2 K show the resistance increase with cooling in this region of temperature, possibly due to presence of a normal segment in the stack.

high-frequency noises propagating along the leads. The measurements were made by using battery-operated low-noise amplifiers (PAR-113). The ramping speeds of the bias current and the threshold voltage (obtained from the maximum voltage of the resistive branch below  $I_{sw}$ ) in measuring the switching current ( $I_{sw}$ ) were  $\dot{I}=30$   $\mu$ A/s and  $V_{th}=110$   $\mu$ V,<sup>18</sup> respectively. Measurements were made on the first jump, which connected the resistive branch to the first QTB of the weakest junction in the stack. For each distribution, 10 000 switching events were recorded with the current resolution of 90 pA.

### III. RESULTS AND DISCUSSION

Figure 1(a) shows the  $R$  vs  $T$  curve of our stacked junction device, which reveals a superconducting transition at  $T_c=90$  K as indicated by the right arrow. The  $c$ -axis tunneling nature in the Bi-2212 stacked junctions is evident by the increasing resistance with decreasing temperature above  $T_c$ . The resistance in the temperature range  $54$  K  $< T < T_c$  origi-

nates from weakly superconducting layers,<sup>19</sup> which have a suppressed critical temperature due to a degradation caused by the high-intensity ion beam in the FIB process. The critical temperature of these layers is about 54 K as shown by the left arrow. The resistance at  $T < 54$  K, however, does not vanish completely, even down to  $T=10$  K, which is attributed to the phase diffusion in the weakened-superconductivity junctions and/or presence of nonsuperconducting junctions in the stack. Figure 1(b) shows the  $V$ - $I$  curves at various temperatures below  $T=54$  K. The nonlinear curve at  $T=50$  K begins to show a hysteresis below  $T \sim 30$  K. At current lower than  $I_{sw}$  the resistance of the resistive branch decreases with decreasing temperature, where the finite resistance is caused by the phase diffusion.<sup>3</sup> The inset of Fig. 1(b) shows the hysteretic  $V$ - $I$  curves at  $T=1.8$ , 3.3, and 4.2 K. Contrary to the  $V$ - $I$  curves in Fig. 1(b), which show a positive curvature below  $I_{sw}$  representing the PDB, the  $V$ - $I$  curves in the inset are characterized by small but negative curvatures. The resistance of the branch below  $I_{sw}$  even increases with decreasing temperature. We speculate that these behaviors should originate from nonsuperconducting layers, which may be strongly damaged by FIB. A similar behavior is observed, e.g., at  $T > T_c$  in Fig. 1(a). In this situation, it is unclear whether the resistive branches are influenced significantly by the phase diffusion or not. On the other hand, since the energy of the phase particle for escaping by thermal fluctuations is dissipated through the environment in the phase-diffusion regime, the switching becomes sensitive to the dissipation process. Therefore, the switching statistics could provide unambiguous information on the role of dissipation, which is not provided by the resistive branch below  $I_{sw}$  at  $T < 4.2$  K in our sample.

Now, we turn to the switching event from the resistive branch for bias current below  $I_{sw}$  to the QTB. Figure 2(a) shows the SWCD (scattered symbols) at various temperatures. All the observed SWCDs are almost symmetric, which is in contrast to asymmetric distributions as predicted for the thermally-activated escape.<sup>6</sup> The standard deviation ( $\sigma$ ) and the mean switching currents ( $\langle I_{sw} \rangle$ ) are shown as a function of temperature in the inset of Fig. 2(a). The temperature dependence of  $\sigma$  contradicts that of a conventional underdamped JJ, where  $\sigma$  increases with temperature in a thermal-activation regime.<sup>2</sup> Furthermore,  $\sigma(T)$  shows a saturation behavior with increasing temperature. Figure 2(b) shows the switching rate,  $\Gamma_S$  (scattered points) vs  $I$  calculated from the SWCD of Fig. 2(a) following the Fulton and Dunkleberger analysis.<sup>2</sup> With lowering temperature,  $\Gamma_S(I)$  shows a pronounced change from an almost linear to a nonlinear behavior with negative curvature in bias-current dependence in a semilogarithmic plot. The same trend in  $\sigma(T)$  has been observed in an IJJ of the same Bi-2212 material near  $T_c$ .<sup>6</sup> In that case,  $I_{sw}$  was almost equal to  $I_r$  and it was assumed that the retrapping process from the QTB to the supercurrent branch affects the switching events. In our case, however, it does not appear plausible that the retrapping process directly affects the switching event because  $I_{sw}$  and  $I_r$  are far from each other, for example,  $I_{sw} \sim 600$  nA and  $I_r \sim 60$  nA at  $T=1.8$  K [inset of Fig. 1(a)].

To explain this behavior in the switching rate, we adopted the multiple-retrapping model developed in Ref. 10 includ-

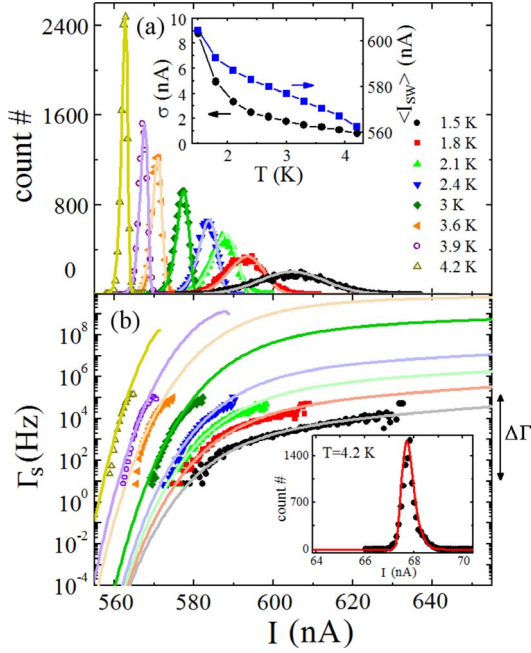


FIG. 2. (Color online) (a) The SWCD and (b) the corresponding  $\Gamma_S$  at various  $T$  from 1.5 to 4.2 K. The solid lines are calculated based on the multiple-retrapping model. Inset of (a): temperature dependence of the standard deviation ( $\sigma$ ) and the mean switching current ( $\langle I_{sw} \rangle$ ).  $\Delta\Gamma$  is the range of the observable switching rate for a given  $I_b$ . Inset of (b): the re trapping current distribution (solid circles) and the calculated behavior (solid line) at  $T=4.2$  K.

ing the effects of junction impedance and temperature. In a phase-diffusion regime, the successive retrapping processes suppress the switching rate,  $\Gamma_S$ . The switching to the high-voltage branch (i.e., the QTB) occurs only when the phase particle is not retrapped after escaping from a local potential minimum. A phase particle that escaped from a potential minimum has a probability,  $P_{RT}$ , to be retrapped in the next potential minimum. The switching rate  $\Gamma_S$ , including  $P_{RT}$ , is expressed by<sup>10</sup>

$$\Gamma_S = \Gamma_{TA}(1 - P_{RT}) \frac{\ln(1 - P_{RT})^{-1}}{P_{RT}}. \quad (1)$$

Here,  $\Gamma_{TA} = \frac{\omega_p}{2\pi} \exp(-\frac{\Delta U}{k_B T})$  is the thermally-activated escape rate,  $\omega_p = \omega_{p0}(1 - \gamma^2)^{1/4}$ ,  $\omega_{p0} = (2eI_c / \hbar C)^{1/2}$ ,  $\Delta U(\phi) \{= 2E_J [(1 - \gamma^2)^{1/2} - \gamma \arccos \gamma]\}$  is the escape energy barrier, and  $\gamma [= I/I_c]$  is a normalized bias current. The retrapping probability can be obtained by an integration (see Ref. 20) of the retrapping rate  $\Gamma_{RT} = \frac{1}{Z_J C} (\Delta U_{RT} / \pi k_B T)^{1/2} \exp(-\Delta U_{RT} / k_B T)$ , where  $\Delta U_{RT}(I) = Z_J^2 C (I - I_{r0})^2$  and  $I_{r0}$  is the noise-free return current from a QTB to a PDB.<sup>21</sup> Here,  $Z_J$  is the fitting parameter representing the total shunt impedance. Figure 3(a) shows an experimental switching distribution in Fig. 2(a) (red dots) at  $T=1.5$  K with the corresponding fit (blue curve) obtained by using Eq. (1) with the best-fit parameters of  $Z_J = 61.9 \Omega$ ,  $I_c = 1.26 \mu A$ , and  $I_{r0} = 63$  nA. The junction capacitance, 330 fF, was estimated from the typical value of 45 fF/ $\mu m^2$  for Bi-2212 IJJs.<sup>22</sup> The corresponding switching rate (red dots) and the fit (blue curve) are shown in Fig. 3(b)

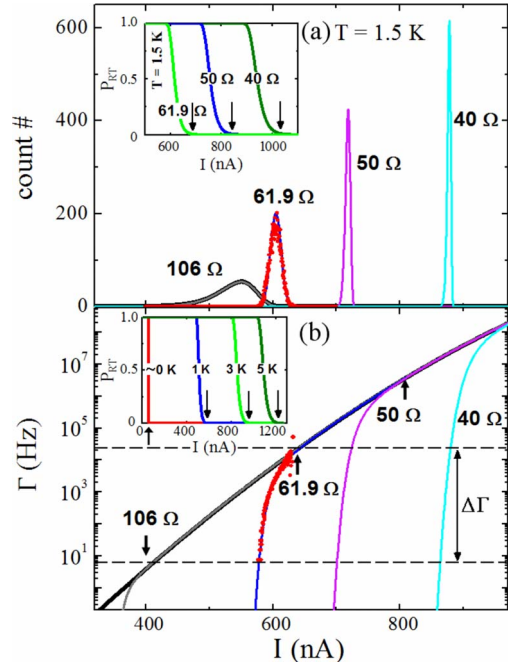


FIG. 3. (Color online) (a) The solid lines represent the calculated SWCDs for various  $Z_J$  values of 106, 61.9, 50, and 40  $\Omega$ , corresponding to  $Q_{PD} = 3.88, 2.37, 1.90,$  and 1.55, respectively. Scattered dots show the SWCD at  $T=1.5$  K. Inset: theoretical re trapping probabilities versus bias current for various  $Z_J$  values at  $T=1.5$  K. The downward arrows indicate the  $I_{PD}$  values. (b) Estimated switching rate versus the bias current. Red dots correspond to the experimental switching rate for  $T=1.5$  K. Estimated  $\Gamma_S$  for various  $Z_J$  values in Eq. (1) are shown as solid lines. Inset: estimated re trapping probabilities versus bias current with  $Z_J = 61.9 \Omega$  at various  $T$ . Thick black curves in (a) and (b) show the thermally-activated SWCD and the corresponding escape rate, respectively, without multiple-retrapping processes.

with the same parameters. An excellent agreement is obtained in both fittings.

These results are analyzed in terms of the  $Z_J$  and  $T$  dependences of  $P_{RT}$ . The inset of Fig. 3(a) shows the calculated  $P_{RT}$  vs  $I$  curves for various  $Z_J$  at  $T=1.5$  K, with the values of  $I_c$  and  $I_{r0}$  obtained from the best fits shown in Fig. 3. The retrapping-probability curve shifts to higher currents as  $Z_J$  decreases due to the  $Z_J$  dependence of  $\Delta U_{RT}$  in the exponential factor of  $\Gamma_{RT}$ . The bias-current positions of almost vanishing  $P_{RT}$ , indicated by downward arrows, approximately correspond to the maximum current allowing the retrapping. We denote this current as  $I_{PD}$ . Physically this is the same current as the one denoted as  $I_m$  in Ref. 11. The system can hardly be retrapped at a current higher than  $I_{PD}$  because in this case the energy fed to the system by the bias current gets larger than the dissipated energy. By equating the energy fed and the energy dissipated, similar to McCumber and Stewart's analysis,<sup>5,11</sup> one obtains the relation

$$I_{PD} = 4I_c / \pi Q_{PD}, \quad (2)$$

where  $Q_{PD}$  is the phase-diffusion quality factor at  $\omega \sim \omega_p$ . In fact, the noise-free retrapping current can be written in a form similar to Eq. (2), namely, as  $I_{r0} = 4I_c / \pi Q(\omega=0)$ .<sup>3</sup> The



TABLE I. Fitting parameters for the switching events for selected temperatures.

$T$ (K)	$I_c$ ( $\mu\text{A}$ )	$I_{r0}$ (nA)	$Z_J$ ( $\Omega$ )	$I_{PD}$ (nA)	$Q_{PD}$
1.5	1.263	63.00	61.9	678	2.37
2.4	1.209	65.10	77.6	683	2.25
3.0	1.006	63.44	86.0	685	1.87
3.6	0.735	65.22	94.7	682	1.37
4.2	0.572	63.79	101.5	572	1.27

thick black curves in Figs. 3(a) and 3(b) show theoretical SWCD and the corresponding  $\Gamma_{TA}(I)$ , respectively, predicted by the thermal-activation model. Other solid curves in Fig. 3(b) are  $\Gamma_S(I)$  [Eq. (1)] for varying  $Z_J$  under the multiple-retrapping processes.  $\Gamma_S(I)$  with each  $Z_J$  in Fig. 3(b) starts to drop quickly from  $\Gamma_{TA}(I)$  at  $I=I_{PD}$ , which are denoted by arrows in the figure. We define  $I_{PD}$  as the bias-current value corresponding to  $P_{RT}=0.01$ , where  $\Gamma_S(I_{PD})$  is nearly the same as the  $\Gamma_{TA}(I_{PD})$  as shown in Fig. 3(b).<sup>10</sup> The impedance of 106  $\Omega$  gives the same SWCD as the thermally-activated SWCD without retrapping because  $\Gamma_S(I)$  overlaps with  $\Gamma_{TA}(I)$  in the observable switching rate window,  $\Delta\Gamma$  [between the two horizontal dashed lines in Fig. 3(b)], although  $Z_J$  is of the order of  $Z_L$ . When  $\Gamma_S(I_{PD})$  start to deviate from the  $\Gamma_{TA}$  in  $\Delta\Gamma$  while  $Z_J$  keeps decreasing, the retrapping phenomenon, which is due to the high-frequency dissipation, begins to influence the SWCD. The observable window of  $\Delta\Gamma$  for a fixed  $\dot{I}$  shifts to the steeper section with decreasing  $Z_J$  or increasing  $I_{PD}$ . This gives a decrease in the SWCD width at a constant temperature [Fig. 3(a)]. We also note that as  $Z_J$  becomes smaller,  $I_{PD}$  becomes larger and correspondingly, according to Eq. (2),  $Q_{PD}$  becomes smaller. Thus, we conclude that the distribution width becomes smaller if the quality factor is reduced.

Since  $I_{PD}$  is also affected by temperature, the shape of  $\Gamma_S(I)$  also depends on temperature. The inset of Fig. 3(b) illustrates  $P_{RT}$  vs  $I$  at various temperatures. Here,  $Z_J$  is fixed at 61.9  $\Omega$ , and other parameters, except for  $T$ , are set to be the same as for the inset of Fig. 3(a). The bias-current position of the zero-temperature curve, indicated by an upward arrow in the inset of Fig. 3(b), corresponds to the fluctuation-free return current  $I_{r0}$ . The value of  $I_{PD}$  shown by arrows increases with increasing temperature. Thus, in effect, the retrapping-probability curve shifts to higher currents as the temperature is raised, as shown in the inset of Fig. 3(b), due to the presence of a Boltzmann-type exponential factor in the expression of  $\Gamma_{RT}$ . To explain the observed SWCD and the corresponding switching rate at different temperatures, therefore, one should consider both effects of the junction shunt impedance and the temperature on the switching events.

Figure 2 illustrates the calculated SWCD and  $\Gamma_S(I)$  as solid curves at various temperatures with the best-fit parameters listed in Table I. The fitting curves agree well with the data. Here, since the temperature is already fixed at the independently measured bath temperature, the main fit parameter becomes  $Z_J$  as in Fig. 3(a). As shown in Fig. 2(b), with

increasing temperature, the calculated  $\Gamma_S(I)$  in the window of  $\Delta\Gamma$  shows steeper regions at higher temperature, which explain the observation of narrower distributions at higher temperatures. Table I shows that the ratio of  $I_c$  and  $I_{PD}$  becomes smaller with increasing temperature.<sup>23</sup> This behavior leads to the conclusion that  $Q_{PD}$  decreases with increasing temperature, following Eq. (2) despite the increase in  $Z_J$  with temperature.<sup>24</sup> This is due to the fact that  $Q_{PD}$  depends on  $I_c$ , which is decreased with increasing temperature.  $I_{PD}$  even equals  $I_c$  at  $T=4.2$  K. This situation is physically the same to an overdamped junction, where  $I_c$  is equal to  $I_{r0}$ . The slope of the calculated  $\Gamma_S(I)$  in the window of  $\Delta\Gamma$  at  $T>4.2$  K is weakly sensitive to temperature variations. This causes the apparent saturation of  $\sigma$  with increasing temperature near  $T=4.2$  K [the inset of Fig. 2(a)]. It should be noticed that at  $T>4.2$  K, the thermal activation causes the escape of the phase particle at the rate of  $\sim 10^7$  Hz for a near-zero bias current. According to the purely thermal-activation model the junction is supposed to be in the phase-run-away state even for zero bias current for this temperature region. The multiple-retrapping process, however, prevents this situation and makes the junction remain in the superconducting state for  $I<I_c$ , accompanied by the PDB as shown in Fig. 2(b). The thermal-activation model shows that this strong phase-diffusion regime exists down to  $T\sim 3$  K. Therefore, we believe that the resistance of the resistive branch below  $I_{sw}$  in this temperature region of the inset of Fig. 1(b) is partially due to the phase diffusion with multiple-retrapping processes.

Finally, we comment on the retrapping dynamics from the QTB to the PDB while reducing the bias current in the quasiparticle-tunneling branch. It is well known that the zero-frequency dissipation plays a significant role in this retrapping process.<sup>11</sup> To explore this zero-frequency damping effect on this system, we obtained the retrapping current distribution from the QTB-PDB switching at  $T=4.2$  K. The inset of Fig. 2(b) shows the stochastic nature of the retrapping current as shown by solid circles at  $T=4.2$  K. The shape of the current distribution shows an asymmetry, i.e., the current region lower than the mean value gives sharper distribution because the noise-free  $I_{r0}$  is a lower bound for the distribution.<sup>7,25</sup> The calculated retrapping distribution (solid line) by the retrapping rate,  $\Gamma_{RT}$ , is consistent with the experimental result. The best-fit parameters are  $I_{r0}=63.8$  nA and  $Z_J=10$  k $\Omega$  ( $\sim R_{qp}$ ), where the estimated noise-free  $I_{r0}$  matches with the value used for  $\Gamma_S$  fitting. The values of  $I_{r0}$  for various temperatures are listed in Table I. The junction shunt impedance  $Z_J$  estimated from the return currents is significantly larger than that found for the switching events. It indicates that the retrapping phenomena from QTB to PDB are mainly determined by a low-frequency limit damping with  $Q(\omega\sim 0)=11.4$  with  $I_c=572$  nA and  $I_{r0}=63.8$  nA at  $T=4.2$  K as shown in Table I.

#### IV. SUMMARY

In this report, we study the high- $T_c$  IJJs, which have suppressed critical currents due to FIB treatment. We show that the multiple-retrapping processes in a hysteretic IJJ with a

high tunneling resistance govern the switching from a resistive state in the phase-diffusion regime into the quasiparticle-tunneling state. The predicted SWCD and  $\Gamma_S$  in the multiple-retrapping model are in good agreement with the observed broadening of the distribution of switching currents with decreasing temperature. We also demonstrate that the change in the shapes of the observed SWCD and  $\Gamma_S$  in various temperatures can be understood by the junction shunt impedance and temperature dependence of the retrapping rate, in terms of the junction quality factor in the phase-diffusion regime. As the macroscopic quantum tunneling has recently been observed in IJJs of Bi-2212 single crystals,<sup>26</sup> this study provides useful information about the dissipative environment.

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